Deformed Two-Mode Quadrature Operators in Noncommutative Space

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Abstract

Starting from noncommutative quantum mechanics algebra, we investigate the variances of the deformed two-mode quadrature operators under the evolution of three types of two-mode squeezed states in noncommutative space. A novel conclusion can be found and it may associate the checking of the variances in noncommutative space with homodyne detecting technology. Moreover, we analyze the influence of the scaling parameters on the degree of squeezing for the deformed level and the corresponding consequences.

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I. INTRODUCTION

Recently quantum mechanics in noncommutative space are widely studied. The main motivation for these theories arises from string theory: the end points of the open strings trapped on an effective D-brane with a nonzero Neveu-Schwarz two form B-field background turn out to be noncommuting [1]. In the presence of the constant antisymmetric tensor field the momentum operators of the D-branes have noncommutative structure. The theoretical and experimental noncommutativity approach in space-space and space-time field to describe the physics at the Planck scale is widely discussed [1–14]. Following the limits of M theory and string theory, the noncommutative gauge theory develops fast, especially noncommutative space quantum field theory by introducing noncommutative quantum mechanics (NCQM) [3–8] has been studied widely, whose applications cover from the Aharonov-Bohm effect to the quantum Hall effect [9–11]. The noncommutative field theories are really challenging because of their nonlocality, which may have consequences on the "CPT theorem" as well as the causality [1–5].

As we all know, near the string scale the space-time appears noncommutativity, that is, the coordinates became no longer commutative. The noncommutative structure in space-time can be introduced in this framework by taking noncommutative coordinates $\hat{x}_\mu$ which satisfy the equation

$$[\hat{x}_\mu, \hat{x}_\nu] = \mathrm{i}\theta_{\mu\nu}, \tag{1}$$

where $\theta_{\mu\nu}$ is a real and antisymmetric tensor with the dimensions of length squared.

In recent years, NCQM has focused on some attention [2–8] such as noncommutative plane, harmonic oscillator, quantum electrodynamics theory, fractional angular momentum in noncommutative space, and constraint on quantum gravitational well etc. The dynamics of charged particle and muliparticle in diverse noncommutative systems with magnetic field or not are deeply discussed for exploring the essentially new features of NCQM. In the researching of two dimensional isotropic harmonic oscillator and testing the spatial noncommutativity via Rydberg atoms, the authors in Refs. [6, 7] from deformed Heisenberg-Weyl Algebra explore the scheme that the consistent ansatz of commutation relations of phase variables should simultaneously include space-space noncommutativity and momentum-momentum noncommutativity. In the following research [8], the author shows that the new type of boson algebra based on the Bose-Einstein statistics at the non-perturbation level described by

deformed annihilation-creation operators is determined by the deformed Heisenberg-Weyl algebra itself, independent of dynamics, where the deformed boson algebra constitutes a complete and closed algebra. The research illuminates that the new type of boson algebra including momentum-momentum noncommutativity is self-contained, and the deformed annihilation and creation operators of the noncommutative space can be represented by the undeformed ones via a linear transformation.

Base on the new type of boson algebra, we investigate the variances of the deformed two-mode quadrature operators under the evolution of three types of two-mode squeezed states in noncommutative space in this paper and we get a novel conclusion. We mark the operator in noncommutative space by using the form adding a hat sign to the common operator $\hat{O}$ in the present paper, so we can introduce the corresponding consistent algebra [6] by

$$\begin{aligned} [\hat{x}_i, \hat{x}_j] = \mathrm{i}\xi^2\theta\epsilon_{ij}\,, \qquad [\hat{x}_i, \hat{p}_j] = \mathrm{i}\hbar\delta_{ij}\,, \\ [\hat{p}_i, \hat{p}_j] = \mathrm{i}\xi^2\eta\epsilon_{ij}, \qquad (i, j = a, b), \end{aligned} \tag{2}$$

where θ and η are the constant parameters, independent of position and momentum; ϵ_{ij} is an two-dimensional antisymmetric unit tensor, $\epsilon_{ab} = -\epsilon_{ba} = 1$ and $\epsilon_{aa} = \epsilon_{bb} = 0$. The scaling factor $\xi = (1 + \theta\eta/4\hbar^2)^{-1/2}$. If momentum-momentum is commuting, then $\eta = 0$, we have $\xi^2 = 1$, the NCQM algebra (2) reduces to the one which is extensively discussed in literature for the case that only space-space are noncommuting.

Now, let us consider the two-mode light quantum field in noncommutative space. Following the realization of the Eq.(2) by deformed variables $\hat{x}_{a(b)}$ and $\hat{p}_{a(b)}$ in Ref [8] and considering light field environment, we choose the consistent ansatz of representations of $\hat{a}(\hat{b}) = \frac{\omega}{\sqrt{2}c}\left(\hat{x}_{a(b)} + \frac{\mathrm{i}c^2}{\hbar\omega^2}\hat{p}_{a(b)}\right)$ and $\hat{a}^\dagger(\hat{b}^\dagger) = \frac{\omega}{\sqrt{2}c}\left(\hat{x}_{a(b)} - \frac{\mathrm{i}c^2}{\hbar\omega^2}\hat{p}_{a(b)}\right)$, where $\hat{a}(\hat{b})$ and $\hat{a}^\dagger(\hat{b}^\dagger)$ are the annihilation and creation operators in noncommutative space. In order to maintain Bose-Einstein statistics at the nonperturbation level [6–8], that is, the operators $\hat{a}^\dagger$ and $\hat{b}^\dagger$ should be commuting to keep the physical meaning, so we have $\eta = \hbar^2\omega^4 c^{-4}\theta$ with the parameter c being velocity of light in vacuum, from Eqs.(2) we can show the commutation relations of $\hat{a}(\hat{b})$ and $\hat{a}^\dagger(\hat{b}^\dagger)$

$$\begin{aligned} [\hat{a}, \hat{a}^\dagger] = [\hat{b}, \hat{b}^\dagger] = 1, \qquad [\hat{a}, \hat{b}] = [\hat{a}^\dagger, \hat{b}^\dagger] = 0, \\ [\hat{a}, \hat{b}^\dagger] = [\hat{a}^\dagger, \hat{b}] = \mathrm{i}\xi^2\omega^2 c^{-2}\theta. \end{aligned} \tag{3}$$

From the relations (2) and (3), we can obtain the annihilation-creation operators [7]. For more physical picture instead of complicated mathematics, we choose $\tan\phi \equiv \omega^2 c^{-2}\theta/2$

and the scaling factor $\xi = (1 + \omega^4 c^{-4}\theta^2/4)^{-1/2} = \cos\phi$, this transformation can simplify the calculation by using the property of the trigonometric function, then we have

$$\begin{aligned} \hat{a} &= a\cos\phi + \mathrm{i}b\sin\phi, & \hat{b} &= b\cos\phi - \mathrm{i}a\sin\phi, \\ \hat{a}^\dagger &= a^\dagger\cos\phi - \mathrm{i}b^\dagger\sin\phi, & \hat{b}^\dagger &= b^\dagger\cos\phi + \mathrm{i}a^\dagger\sin\phi. \end{aligned} \tag{4}$$

The property of single-mode quadrature operators [6] has been studied, one finds that variances of single-mode quadrature operator in one degree of freedom include variances in the other degree of freedom. The result is of importance and inspire us to go one step further to clarify the role of the scaling parameters in the whole process by investigating the features of two-mode quadrature operators in noncommutative space and we can obtain a novel conclusion and it may provide a feasible checking scheme to the noncommutative two-mode squeezed states via homodyne detecting technology.

The paper is organized as follows. In the following section we show variances of the two-mode quadrature operators in commutative space. Then we calculate variances of the two-mode quadrature operators in noncommutative space and compare the corresponding consequences to the commutative case, we find variances of the two-mode quadrature operator in noncommutative space can be written with variances in commutative space and additional "correction terms". Finally, we discuss the corresponding results, analyze the effect of the scaling parameter on the squeezing degree and clarify the possibility of the checking scheme to the noncommutative two-mode squeezed states.

II. TWO-MODE QUADRATURE OPERATORS IN COMMUTATIVE SPACE

In this part, we will show variances of the two-mode quadrature operators in commutative space. Following the former work [15, 16], according to the two-mode quadrature operators in commutative space

$$\begin{aligned} X &= \frac{1}{2\sqrt{2}}\left(a + a^\dagger + b + b^\dagger\right), \\ Y &= \frac{1}{\mathrm{i}2\sqrt{2}}\left(a - a^\dagger + b - b^\dagger\right), \end{aligned} \tag{5}$$

we can study the properties of the two-mode quadrature operators explicitly using a special effective squeezed state presented by J. Janszky and A.V. Vinogradov [17], which can be achieved through superposition of coherent states along a straight line on the α plane. For

a single mode of frequency ω, the electric field operator $E(t)$ is represented as $E(t) = E_0[a\exp(-i\omega t) + a^\dagger \exp(i\omega t)]$, where a and $a^\dagger$ are the annihilation and creation operators of photon field. The superposition states is defined by

$$\begin{aligned} |\alpha, \pm\rangle &= c_{\alpha\pm} \left(|\alpha\rangle \pm |-\alpha\rangle\right), \\ c_{\alpha\pm} &= \{2[1 \pm \exp(-2\alpha^2)]\}^{-1/2}, \end{aligned} \tag{6}$$

where $|\alpha\rangle$ is the usual coherent state and the algebra satisfies $a|\alpha,\pm\rangle = \alpha c_{\alpha\pm} c_{\alpha\mp}^{-1} |\alpha,\mp\rangle$, $a^2|\alpha,\pm\rangle = \alpha^2|\alpha,\pm\rangle$, and $\langle\alpha,\pm|\alpha,\mp\rangle = 0$. For the sake of simplicity, we have assumed that α is real.

In order to compare the results in commutative space with those in the noncommutative space, we will first consider the variances of the two-mode quadrature operators under the evolution of three types of two-mode squeezed states in commutative space. To achieve this goal, below let us construct three types of two-mode squeezed states. From two-mode coherent state $|\alpha\rangle|\beta\rangle$ (the basic algebra $a|\alpha\rangle|\beta\rangle = \alpha|\alpha\rangle|\beta\rangle$, $b|\alpha\rangle|\beta\rangle = \beta|\alpha\rangle|\beta\rangle$), we can define three types of two-mode squeezed states [6, 17], given by

$$\begin{aligned} |\Psi_1, \pm\rangle &\equiv |\alpha, \pm\rangle|\beta\rangle, \\ |\Psi_2, \pm\rangle &\equiv |\alpha, \pm\rangle|\beta, \pm\rangle, \\ |\Psi_3, \pm\rangle &\equiv c_{3\pm} \left(|\alpha, \beta\rangle \pm |-\alpha, -\beta\rangle\right), \end{aligned} \tag{7}$$

where $c_{3\pm}^2 = 1/\{2[1 \pm \exp(-2\alpha^2 - 2\beta^2)]\}$, and the corresponding symbol between the equal mark changes synchronously (e.g., $|\Psi_2, +\rangle \equiv |\alpha, +\rangle|\beta, +\rangle$ and $|\Psi_2, -\rangle \equiv |\alpha, -\rangle|\beta, -\rangle$).

For the sake of describing the squeezing degree of the two-mode squeezed state, we should take account into the uncertainty relation via the variance relation $(\Delta X)^2 = \langle X^2\rangle - \langle X\rangle^2$ and $(\Delta Y)^2 = \langle Y^2\rangle - \langle Y\rangle^2$. For three different squeezed states, we have obtain the corresponding results, and the squeezed state appears only in $|\Psi_i, +\rangle$ (i=1, 2, 3).

First, for the case of $|\Psi_1, +\rangle$, applying the simple relations $\langle\Psi_1, +|ab|\Psi_1, +\rangle = \langle\Psi_1, +|a^\dagger b^\dagger|\Psi_1, +\rangle = 0$ and $\langle\Psi_1, +|a^\dagger b|\Psi_1, +\rangle = \langle\Psi_1, +|ab^\dagger|\Psi_1, +\rangle = 0$, after some algebra, the variance of the two-mode quadrature operator can be put into the following form

$$\begin{aligned} (\Delta X)^2_{\Psi_{1+}} &= \frac{1}{4} + \frac{\alpha^2}{2[1 + \exp(-2\alpha^2)]}, \\ (\Delta Y)^2_{\Psi_{1+}} &= \frac{1}{4} - \frac{\alpha^2}{2[1 + \exp(2\alpha^2)]}, \end{aligned} \tag{8}$$

where the same term $\beta^2/2$ in both $\langle X^2\rangle_{\Psi_{1+}}$, $\langle Y^2\rangle_{\Psi_{1+}}$ and $\langle X\rangle^2_{\Psi_{1+}}$, $\langle Y\rangle^2_{\Psi_{1+}}$ occurs, so the result shows that the variance is independent of parameter β for $|\Psi_1,+\rangle$.

Next, for the case of $|\Psi_2,+\rangle$, according to the same procedure as above, we have the results

$$\begin{aligned}(\Delta X)^2_{\Psi_{2+}} &= \frac{1}{4}+\frac{\alpha^2}{2[1+\exp(-2\alpha^2)]}+\frac{\beta^2}{2[1+\exp(-2\beta^2)]},\\ (\Delta Y)^2_{\Psi_{2+}} &= \frac{1}{4}-\frac{\alpha^2}{2[1+\exp(2\alpha^2)]}-\frac{\beta^2}{2[1+\exp(2\beta^2)]}.\end{aligned} \tag{9}$$

Finally, for the case of $|\Psi_3,+\rangle$, according to the relations $\langle\Psi_3,+|ab|\Psi_3,+\rangle = \langle\Psi_3,+|a^\dagger b^\dagger|\Psi_3,+\rangle = \alpha\beta$ and $\langle\Psi_3,+|a^\dagger b|\Psi_3,+\rangle = \langle\Psi_3,+|ab^\dagger|\Psi_3,+\rangle = \alpha\beta\frac{c_{3+}^2}{c_{3-}^2}$, the variances of X and Y are given by

$$\begin{aligned}(\Delta X)^2_{\Psi_{3+}} &= \frac{1}{4}+\frac{(\alpha+\beta)^2}{2[1+\exp(-2\alpha^2-2\beta^2)]},\\ (\Delta Y)^2_{\Psi_{3+}} &= \frac{1}{4}-\frac{(\alpha+\beta)^2}{2[1+\exp(2\alpha^2+2\beta^2)]}.\end{aligned} \tag{10}$$

III. DEFORMED TWO-MODE QUADRATURE OPERATORS IN NONCOMMUTATIVE SPACE

In this section let us show what is the situation for the two-mode quadrature operators in the noncommutative space. Using the relations (4) and (5), we can construct the deformed two-mode quadrature operators as follows

$$\begin{aligned}\hat{X} &\equiv \frac{1}{2\sqrt{2}}\left(\hat{a}+\hat{a}^\dagger+\hat{b}+\hat{b}^\dagger\right) = X\cos\phi + Y^{ab}\sin\phi,\\ \hat{Y} &\equiv \frac{1}{\mathrm{i}2\sqrt{2}}\left(\hat{a}-\hat{a}^\dagger+\hat{b}-\hat{b}^\dagger\right) = Y\cos\phi + X^{ab}\sin\phi,\\ &[\hat{X},\hat{Y}] = \frac{\mathrm{i}}{2},\end{aligned} \tag{11}$$

where we have introduced the definitions $Y^{ab} = (Y^a - Y^b)/\sqrt{2}$ and $X^{ab} = (X^b - X^a)/\sqrt{2}$ for the sake of convenience. The quantities X^a, X^b, Y^a and Y^b are the corresponding a(b)-mode quadrature operators, denoted by $X^a = \frac{1}{2}(a+a^\dagger), Y^a = \frac{1}{2\mathrm{i}}(a-a^\dagger)$ and $X^b = \frac{1}{2}(b+b^\dagger), Y^b = \frac{1}{2\mathrm{i}}(b-b^\dagger)$, which are frequently used in the squeezed quantum field. The expressions of X and Y are identical to Eqs. (5). It is pointed out that the deformed quadrature operators satisfy the same commuting relation as the case in commutative space.

In what follows we concentrate on the variances of deformed two-mode quadrature operators. Inserting Eqs.(11) into the relations $(\Delta\hat{X})^2 = \langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2$ and $(\Delta\hat{Y})^2 = \langle\hat{Y}^2\rangle - \langle\hat{Y}\rangle^2$, we have

$$\begin{aligned}(\Delta\hat{X})^2 &= \left\langle \left(X\cos\phi + Y^{ab}\sin\phi\right)^2\right\rangle - \left\langle X\cos\phi + Y^{ab}\sin\phi\right\rangle^2,\\ (\Delta\hat{Y})^2 &= \left\langle \left(Y\cos\phi + X^{ab}\sin\phi\right)^2\right\rangle - \left\langle Y\cos\phi + X^{ab}\sin\phi\right\rangle^2,\end{aligned} \tag{12}$$

where $\cos^2\phi = \xi^2 = (1+\omega^4c^{-4}\theta^2/4)^{-1}$ and $\sin^2\phi = 1-\xi^2$ are the θ-dependent experiment parameters [3, 7, 12].

If we expand equations above and do some rearrangement, we can arrive at a novel consequence

$$\begin{aligned}(\Delta\hat{X})^2 &= (\Delta X)^2 + \left[(\Delta Y^{ab})^2 - (\Delta X)^2\right]\sin^2\phi + C_{\Delta\hat{X}},\\ (\Delta\hat{Y})^2 &= (\Delta Y)^2 + \left[(\Delta X^{ab})^2 - (\Delta Y)^2\right]\sin^2\phi + C_{\Delta\hat{Y}},\end{aligned} \tag{13}$$

where $C_{\Delta\hat{X}}$ and $C_{\Delta\hat{Y}}$ represent the cross terms of the Eqs.(12), the specific expression are $C_{\Delta\hat{X}} = (\langle XY^{ab} + Y^{ab}X\rangle - 2\langle X\rangle\langle Y^{ab}\rangle)\sin\phi\cos\phi$ and $C_{\Delta\hat{Y}} = (\langle YX^{ab} + X^{ab}Y\rangle - 2\langle Y\rangle\langle X^{ab}\rangle)\sin\phi\cos\phi$, respectively. The calculation process become compact instead of complicated via introducing the trigonometric function transformation.

From Eqs.(13), we can find variances of the two-mode quadrature operator in noncommutative space can be written with variances in commutative space and additional "correction terms", the scaling parameter has no contribute to the first term in Eqs.(13). It it inspiring, because the quadrature variance of squeezed state can be measured directly via homodyne detection technology [18]. This is a very interesting result for the influence of the noncommutative space to the two-mode squeezing field and it can provide a novel checking scheme to the noncommutative two-mode squeezed states in a simple way. Following the universal equations above, we can get the corresponding results for different type of the two-mode squeezing light field in noncommutative space, respectively.

To begin with, for the case of $|\Psi_1,+\rangle$, after some conversion we have $(C_{\Delta\hat{X}})_{\Psi_1+} = (C_{\Delta\hat{Y}})_{\Psi_1+} = 0$, $(\Delta Y^{ab})^2_{\Psi_1+} = 1/4 - \alpha^2/\{2[1+\exp(2\alpha^2)]\}$ and $(\Delta X^{ab})^2_{\Psi_1+} = 1/4 + \alpha^2/\{2[1+\exp(-2\alpha^2)]\}$. Substituting the above results into Eq. (12), the corresponding variances follow

$$\begin{aligned}(\Delta\hat{X})^2_{\Psi_1+} &= (\Delta X)^2_{\Psi_1+} - \frac{\alpha^2}{2}(1-\xi^2),\\ (\Delta\hat{Y})^2_{\Psi_1+} &= (\Delta Y)^2_{\Psi_1+} + \frac{\alpha^2}{2}(1-\xi^2),\end{aligned} \tag{14}$$

we can find the only modification from noncommutative space is a factor $\alpha^2(1-\xi^2)/2$ here comparing to Eq.(8) for the state vector, here the cross term in Eq.(13) is zero and it is same to state vectors else.

Next, for the case of $|\Psi_2,+\rangle$, after some algebra we have $(C_{\Delta\hat{X}})_{\Psi_2+} = (C_{\Delta\hat{Y}})_{\Psi_2+} = 0$, $(\Delta Y^{ab})^2_{\Psi_2+} = 1/4 - \alpha^2/\{2[1+\exp(2\alpha^2)]\} - \beta^2/\{2[1+\exp(2\beta^2)]\}$ and $(\Delta Y^{ab})^2_{\Psi_2+} = 1/4 + \alpha^2/\{2[1+\exp(-2\alpha^2)]\} + \beta^2/\{2[1+\exp(-2\beta^2)]\}$. As such, we can obtain the corresponding results

$$\begin{aligned}(\Delta\hat{X})^2_{\Psi_2+} &= (\Delta X)^2_{\Psi_2+} - \frac{(\alpha^2+\beta^2)}{2}(1-\xi^2)\\ (\Delta\hat{Y})^2_{\Psi_2+} &= (\Delta Y)^2_{\Psi_2+} + \frac{(\alpha^2+\beta^2)}{2}(1-\xi^2)\end{aligned} \tag{15}$$

the only modification from noncommutative space is a factor $(\alpha^2+\beta^2)(1-\xi^2)/2$.

And for the case of $|\Psi_3,+\rangle$, making full use of $(C_{\Delta\hat{X}})_{\Psi_3+} = (C_{\Delta\hat{Y}})_{\Psi_3+} = 0$, $(\Delta Y^{ab})^2_{\Psi_3+} = 1/4 - (\alpha-\beta)^2/\{2[1+\exp(\alpha^2+\beta^2)]\}$ and $(\Delta X^{ab})^2_{\Psi_3+} = 1/4 + (\alpha-\beta)^2/\{2[1+\exp(-\alpha^2-\beta^2)]\}$ we can obtain corresponding variances

$$\begin{aligned}(\Delta\hat{X})^2_{\Psi_3+} &= (\Delta X)^2_{\Psi_3+} - \left\{\frac{\alpha^2+\beta^2}{2} + \frac{\alpha\beta[1-\exp(-2\alpha^2-2\beta^2)]}{1+\exp(-2\alpha^2-2\beta^2)}\right\}(1-\xi^2),\\ (\Delta\hat{Y})^2_{\Psi_3+} &= (\Delta Y)^2_{\Psi_3+} + \left\{\frac{\alpha^2+\beta^2}{2} + \frac{\alpha\beta[1-\exp(-2\alpha^2-2\beta^2)]}{1+\exp(-2\alpha^2-2\beta^2)}\right\}(1-\xi^2).\end{aligned} \tag{16}$$

the only modification from noncommutative space is a term depending on the scaling parameter too.

Equations (14)-(16) are the main results for the two-mode quadrature operators under the evolution of three types of two-mode squeezed states in noncommutative space, they all come from the universal Equations (13). We can compare these results to the commutative case and analyze the influence of the scaling parameters on the degree of squeezing for the deformed level and the corresponding consequences.

IV. DISCUSSIONS AND CONCLUSIONS

There are different bounds on the parameter θ set by experiments. Although θ is surely small, the existing experiments [3, 7, 12–14] demonstrate the approximate order of magnitude. The low-energy test[12] of Lorentz invariance place bounds on noncommutative energy scale of order 10 Tev. Measurements of the Lamb shift [13] give a weaker bound. Far stronger

bounds arise if one considers potential noncommutative effects in strong interactions[14]. For the scaling factor $\xi^2 = \left(1+\omega^4 c^{-4}\theta^2/4\right)^{-1}$, from the specific experimental parameter we can analyze the scaling factor in detail. On the other hand, detection and measurement of squeezed states via homodyne detection is a proven technology [18]. It may be a feasible scheme to study NCQM utilizing the auxiliary deformed two-mode squeezed state. In order to associate the variances with the experiment parameter θ, we expand the scaling factor ξ^2 with respect to θ using Taylor power series

$$\xi^2 = 1 - A\theta^2 + O(\theta^4), \tag{17}$$

where $A = \omega^4 c^{-4}/4$, and we only retain the terms up to the second order of the parameter θ. Inserting the Taylor expansion (17) into Eqs. (14)-(16), the influence of the noncommutativity on the two-mode quadrature operators can be determined by

$$\begin{aligned}(\Delta\hat{X})^2_{\Psi_{1+}} &= (\Delta X)^2_{\Psi_{1+}} - \frac{\alpha^2}{2}A\theta^2,\\ (\Delta\hat{Y})^2_{\Psi_{1+}} &= (\Delta Y)^2_{\Psi_{1+}} + \frac{\alpha^2}{2}A\theta^2,\end{aligned} \tag{18}$$

$$\begin{aligned}(\Delta\hat{X})^2_{\Psi_{2+}} &= (\Delta X)^2_{\Psi_{2+}} - \frac{\alpha^2+\beta^2}{2}A\theta^2,\\ (\Delta\hat{Y})^2_{\Psi_{2+}} &= (\Delta Y)^2_{\Psi_{2+}} + \frac{\alpha^2+\beta^2}{2}A\theta^2,\end{aligned} \tag{19}$$

$$\begin{aligned}(\Delta\hat{X})^2_{\Psi_{3+}} &= (\Delta X)^2_{\Psi_{3+}} - \left\{\frac{\alpha^2+\beta^2}{2} + \frac{\alpha\beta[1-\exp(-2\alpha^2-2\beta^2)]}{1+\exp(-2\alpha^2-2\beta^2)}\right\}A\theta^2,\\ (\Delta\hat{Y})^2_{\Psi_{3+}} &= (\Delta Y)^2_{\Psi_{3+}} + \left\{\frac{\alpha^2+\beta^2}{2} + \frac{\alpha\beta[1-\exp(-2\alpha^2-2\beta^2)]}{1+\exp(-2\alpha^2-2\beta^2)}\right\}A\theta^2.\end{aligned} \tag{20}$$

From Eqs. (18)-(20), we can arrive at the conclusion that the degree of the squeezing in noncommutative space becomes attenuated explicitly in Y direction and the degree of the squeezing in X direction becomes enhanced, as compared to that in commutative space. When $\theta = 0$, it is straightforward to show from Eqs. (14)-(16) or Eqs. (18)-(20) that the variances in noncommutative space turn back into the ones in commutative space, which is in good agreement with the commutation relations (3). Alternatively, when the noncommutative system is determinate, the variances of the two-mode quadrature operators under the evolution of three types of two-mode squeezed states can be written in the form of the simple

addition or subtraction with the variances in commutative condition and some "correction terms" connected with experiment parameter θ.

In summary, starting from noncommutative quantum mechanics algebra, we investigate the variances of the two-mode quadrature operators under the evolution of three types of two-mode squeezed states in commutative and noncommutative space, and point out the difference between them. In addition, we analyze the influence of the scaling parameters on the degree of the squeezing for the deformed level and illuminate the corresponding conclusions, which may be helpful for testing the uncertainty relation and the essential features of the noncommutative space in corresponding experiments. Our results are of theoretical interest in understanding the characteristics of the two-mode squeezing light fields in noncommutative space, and have potential application in searching new schemes for the relating experimental testing of noncommutative space.

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